International Journal of Theoretical Physics, Vol. 28, No. 9, 1989

First Flash and Second Vacuum

David Finkelstein[1]

Received June 8, 1989

I propose that history is a network of quantum jumps. The universe begins as a single point. It multiplies by fission into a chaotic but causally ordered phase of indeterminate dimensionality. This in turn condenses into our ambient vacuum. Only then does inertia arise, as a macroscopic quantum effect.

1. INTRODUCTION

The question that opens this volume is whether the origin of the universe is a singularity or instability. The answer I would like to put forward is "No." For to call the origin a singularity would be a euphemism for saying that our theory does not in fact describe it, and to call it an instability would imply that there is a pre-existing system to be unstable. Both singularity and instability are smoothed continuum descriptions of important phases of the early history of the universe, but neither is a true origin. It seems unlikely that any theory based on a space-time continuum can describe the origin of that continuum. It happens that the algebraic theory I am currently developing provides a true origin for the universe.

Even the first and most primitive theory of what came to be called a black hole showed that sufficiently extended gravitational sources may be at spacelike separations from a remote observer, but sufficiently concentrated spherical sources, and as an extreme case singular point sources, lie either in the past or the future of the observer, on the far side of a "unidirectional membrane" (Finkelstein, 1958). Such singularities, where the differential manifold theory of space-time breaks down, seem to represent the birth pangs of the universe and the death pangs of stars (Ruffini and Wheeler, 1970). To regularize them and bring them into the scope of physics, I have been developing a quantum-algebraic theory of space-time. Since classical topology is based on classical set theory, it seems natural to base quantum topology on a quantum set theory (Finkelstein *et al.*, 1959). It is reviewed, updated, and applied to cosmogony in this paper.

[1]Georgia Institute of Technology, Atlanta, Georgia 30332.

0020-7748/89/0900-1081$06.00/0

Others have considered such theories:

> To be sure, it has been pointed out that the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature, that is, to the elimination of continuous functions from physics. Then, however, we must also give up, by principle, the space-time continuum. It is not unimaginable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At the present time, however, such a program looks like an attempt to breathe in empty space. (Einstein, 1936)

> It may well be that the marriage of gravitation and quantum mechanics requires a few more drastic revisions of our ideas. For example, our description of space-time as a continuum may have to be replaced by a discrete granular structure at extremely short distance. (National Research Council, 1986*a*)

> It may be that local Lagrangian field theory is not the correct approach to quantum gravity. Perhaps, as some believe, the basic quantum quantities are not the variables describing a space-time continuum but a more discrete structure. (National Research Council, 1986*b*)

QND (quantum network dynamics) is a "purely algebraic method of description of nature" as a network of quantum jumps. The language of QND permits us to formulate the creation of the universe in at least the following stages:

- The First Flash. The point origin of the universe and time (Finkelstein, 1969). This is not quite the vacuum fluctuation of Tryon (1973), which assumes a preexistent space-time continuum and Hamiltonian; nor the pregeometry of Wheeler (1973) and Misner *et al.* (1973).
- Vacuum II. A chaotic but causally ordered phase of QND, a tangle of arrows of time.
- Vacuum I. A coherent state of QND simulating a manifold [Finkelstein (1978, esp. p. 13; 1987, 1988, 1989). Chew and Stapp (1989) also propose that space-time is a coherent state].

2. QUANTUM-SPACE-TIME

In QND space-time does not act on matter nor matter on space-time. There neither has separate existence. Both are smoothed descriptions of the quantum-space-time dynamical network, which acts upon itself.

Standard quantum theory is a penthouse added to an already completed tower of classical physics (Figure 1). QND began as a plan to rebuild this tower on quantum foundations and simplified into a one-level structure (Figure 2) based on the scalars 0 and 1, a partial operation $\vee$, and two operations ι and $+$. Dynamical succession is represented by ι, multiplicity

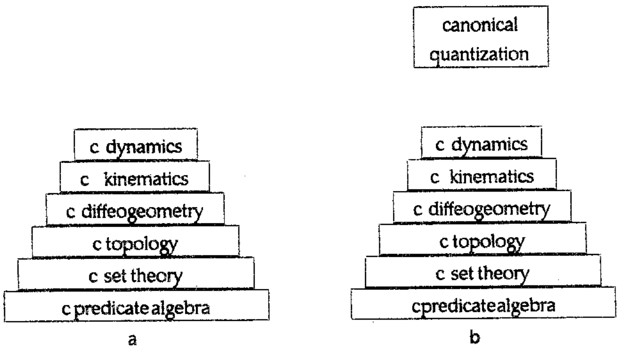

Fig. 1. Architectures of classical and quantum field theory. Canonical quantization is added to a finished classical field theory (a) to make quantum field theory (b).

by $\vee$, and quantum superposition by $+$. For didactic reasons I introduce ι, $\vee$, and $+$ one by one here.

2.1. Time

Peano's theory of the natural numbers $\mathbb{N}$ postulates the existence of 1, a mapping ι called successor, and $\mathbb{N}$. When I interpret $\mathbb{N}$ physically as a discrete time I shall write it as $\mathbb{T}$. Peano's postulates are

T1. $1 \in \mathbb{T}$.
T2. $\alpha \in \mathbb{T} \Rightarrow \iota\alpha \in \mathbb{T}$.
T3. $1 \notin \iota'\mathbb{T}$.
T4. $\mathbb{T}$ is the least class satisfying T1–T3.

[The prime in T3 and below lifts a mapping from idividuals to sets; for any subset $S \subset \mathbb{T}$, one writes "the ι's of S" as

$$\iota' S := \{\iota\sigma \mid \sigma \in S\}$$

and similarly for $\vee'$.]

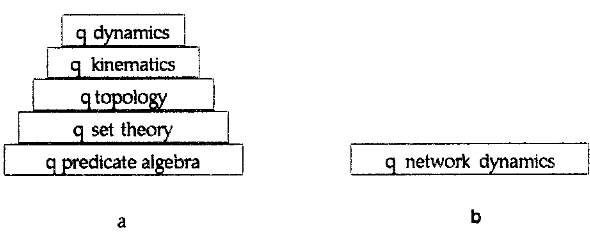

Fig. 2. Architectures of QND. When the principle of superposition is moved from the top of Fig. 1b to the bottom of part (b) of the present figure and the successor operator ι is introduced, the tower telescopes into one story (b).

T1–T4 amount to a recursive scheme for generating $\mathbb{T}$ that emulates the passage of time:

$$\mathbb{T} = \{1\} \cup \iota'\mathbb{T}$$

$\{1\}$ is the set whose only element is 1. Each step of the recursion consists in inserting a previously defined approximation to $\mathbb{T}$ into the right-hand side and taking the right-hand side as the next approximation. The initial $\mathbb{T}$ is the null set. The limit $\mathbb{T}$ is the infinite sequence

$$\mathbb{T} = \{1, \iota 1, \iota\iota 1, \ldots\}$$

Thus, the events of $\mathbb{T}$ are words in two symbols; 1 expresses the beginning and ι the advance of time.

2.2. Space-Time

To model space and its contents as well as time, I extend $\mathbb{T}$ to a tree $\mathbb{S}$ of higher dimensional patterns of dynamical succession. I do not define a space-time and then attach fields to it; I define the network and abstract space-times and fields from it. It would be a mistake to seek a resemblance to Minkowski space-time prior to the quantum theory.

Let $\vee$ be the partial operation of disjoint union, defined only for disjoint sets and then agreeing with the union. 1 is the identity of this partial operation. I postulate the existence of $\mathbb{S}$, $1 \in \mathbb{S}$, $\iota: \mathbb{S} \to \mathbb{S}$, and $\vee: \mathbb{S} \times \mathbb{S} \to \mathbb{S}$, with

$$\alpha \vee (\beta \vee \gamma) = (\alpha \vee \beta) \vee \gamma, \qquad \alpha \vee \beta = \beta \vee \alpha, \qquad 1 \vee \alpha = \alpha,$$

$$0 \vee \alpha = \iota\alpha \vee \iota\alpha = \iota 0 = 0$$

So $\vee$ is an associative commutative nilpotent product with identity 1 (the null set) and zero 0 (the undefined set); The appropriate replacement for Peano's postulates T1–T4 is now:

S1. $1 \in \mathbb{S}$.
S2(ι). $\alpha \in \mathbb{S} \Rightarrow \iota\alpha \in \mathbb{S}$.
S2($\vee$). $\alpha \in \mathbb{S}$ and $\beta \in \mathbb{S} \Rightarrow \alpha \vee \beta \in \mathbb{S}$.
S3. $1 \notin \iota'\mathbb{S}$.
S4. $\mathbb{S}$ is the least class satisfying S1–S3.

The recursion for S1–S4 is then

$$\mathbb{S} = \{1\} \cup \iota'\mathbb{S} \cup \mathbb{S} \vee{}'\mathbb{S}$$

Peano's operator $\iota\alpha$ is the operator $\{\alpha\}$ of forming a unit set or monad. $\mathbb{S}$ is the space of all finitistic classical sets.

As usual, an element of $\mathbb{S}$ may be interpreted either as predicate or as set. For predicates, 1 is usually written F or FALSE and $\alpha \vee \beta$ is the partial OR of Peirce (1886*a,b*), written $\alpha \text{POR} \beta$ here. In terms of more familiar predicate operations,

$$\alpha \text{POR} \beta = \alpha \text{OR} \beta \qquad \text{if} \quad \alpha \text{AND} \beta = \text{F}$$
$$\alpha \text{POR} \beta = 0 \qquad \text{otherwise}$$

Here "$=0$" means "is undefined." OR and XOR are complete operations, POR is partial. Elsewhere I write SET for $\mathbb{S}$.

An element η of $\mathbb{S}$ can also be read as a network history. The elements $\eta_1, \ldots, \eta_N$ of η represent the final events of that history. The elements $\eta_{11}, \ldots, \eta_{1M}$ of η_1 represent the immediate antecedents of η_1, and so forth, back to 1. The operator ι then represents dynamical succession, and each event is an ancestral element of all its antecedents.

$\mathbb{S}$ is the space of all ancestrally finite sets, a universal language for finite mathematical objects, and can represent any finite dynamical process. The unit of such a process is a dynamical event α, which is maximally local and then maximally informative, corresponding to a space-time point with its basic field variables but not their derivatives. These make up a graph $E \subset \mathbb{S}$. The vertices α of E represent dynamical events. The edges δ of E replace directional derivatives. [The approximation of Minkowskian manifolds by such graphs is treated, for example, by Bombelli *et al.* (1987).] The basic principle connecting events is neither a topology nor a causal relation, but the dynamical successor operator ι.

A kinematically possible history expresses each event α' by means of ι in terms of its predecessors α and links δ. Since the same event generally has various representations of this kind, it is generally necessary to form equivalence classes. These form a subspace $KH \subset \mathbb{S}$.

A dynamical history is the submanifold of KH obeying the dynamical law DL of the theory. The function space of all such histories is the space DH of dynamical histories. DH is the phase space of the system (Barrett, 1989). Thus, DH is the subspace of KH subject to the dynamical law DL,

$$DH = KH \backslash DL \subset \mathbb{S}$$

Thus, one can represent any discrete classical dynamical process exactly by a quotient space of a subspace of $\mathbb{S}$, keeping the dynamical interpretation of ι; and can represent any continuous process similarly, therefore, as closely as one likes. I illustrate this construction below.

2.3. Quantum-Space-Time

$\mathbb{Q}$ is a universal quantum-space-time in the sense that $\mathbb{S}$ is a universal classical space-time. Besides the space-time concepts of $\mathbb{S}$, it has a quantum superposition operation +. Since $\vee$ is the only product that occurs, I usually write $\alpha \vee \beta$ as $\alpha\beta$. I postulate the existence of

$$\mathbb{Q}$$
$$1 \in \mathbb{Q}$$
$$\iota: \quad \mathbb{Q} \to \mathbb{Q}$$
$$\vee: \quad \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$$
$$+: \quad \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$$

and that $(\mathbb{Q}, \vee, +)$ is a Grassmann ring over $\iota'\mathbb{Q}$:

$$\alpha(\beta\gamma) = (\alpha\beta)\gamma, \qquad 1\alpha = \alpha = \alpha 1, \qquad 0\alpha = 0, \qquad (\iota\alpha)(\iota\alpha) = 0$$
$$\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma, \qquad \alpha + \beta = \beta + \alpha, \qquad 0 + \alpha = \alpha, \qquad \alpha + (-\alpha) = 0$$
$$\iota(\alpha + \beta) = \iota\alpha + \iota\beta, \qquad \iota(-\alpha) = -\iota\alpha$$

Note that monads, which commute in $\mathbb{S}$, anticommute in $\mathbb{Q}$. I call its elements quantum sets or qets. Elsewhere I write QET for $\mathbb{Q}$.

The scheme for generating events replacing T1–T4 and S1–S4 is now:

Q1. $1 \in \mathbb{Q}$.
Q2(ι). $\alpha \in \mathbb{Q} \Rightarrow \iota\alpha \in \mathbb{Q}$.
Q2($\vee$). $\alpha \in \mathbb{Q}$ and $\beta \in \mathbb{Q} \Rightarrow \alpha \vee \beta \in \mathbb{Q}$.
Q2(+). $\alpha \in \mathbb{Q}$ and $\beta \in \mathbb{Q} \Rightarrow \alpha - \beta \in \mathbb{Q}$.
Q3. $1 \notin \iota'\mathbb{Q}$.
Q4. Any class satisfying Q1–Q3 includes $\mathbb{Q}$.

The recursion is now

$$\mathbb{Q} = \{1\} \cup \iota'\mathbb{Q} \cup \mathbb{Q} \vee '\mathbb{Q}. \cup \mathbb{Q} - '\mathbb{Q}$$

It is useful to write $\iota\alpha$ as a qet bracket $\langle\alpha|$, uniting the ket bracket $|\alpha\rangle$ of Dirac, the set bracket $\{\alpha\}$ of Cantor, and the successor operator ι of Peano:

$$\iota\alpha =: \langle\alpha| = \{\alpha\}$$

Qets nest like sets, add like kets, and multiply like Grassmann extensions. They represent either quantum sets or quantum predicates.

$\mathbb{Q}$ serves as the space of quantum kinematical histories KH of a universal finitistic quantum entity and its Fermi–Dirac ensembles. First-grade elements of $\mathbb{Q}$, or monads, represent dynamical events; ι, dynamical succession; and $\vee$, combination. The product $\alpha\beta$ unites the Grassmann product of fermionic quantum kinematics, the disjoint union of set theory, and the Peirce partial operation POR. Equivalence classes define kinematical histories as with $\mathbb{S}$. Now the quantum-space-time is also a one-quantum phase space, in the sense that a momentum state is a superposition of position states.

I omit the complex numbers $\mathbb{C}$ from the foundations because we can make them from what we have. A complex number is a pair of reals; a real is physically indiscernible from a rational; rational vector components may be turned into integers in a finite-dimensional vector space by rescaling the state vector, with no change in its physical meaning; and we can make the integers with 1, +, and −. The imaginary unit i is then an operator associated with a superselection law arising from random phases. I use a provisional i in the present work for the coefficients of SL_2.

Set theory is said to be a universal language for mathematics. Nevertheless, the quantum physical world cannot be represented in classical set theory in the strong sense I use here, but possibly can in quantum set theory. I give each of the primitives $\{1, \iota, \vee, +\}$ a single uniform physical meaning (and 0 none at all). That is, 1 expresses the beginning, ι temporal succession, $\vee$ the combination of coexistent alternatives, and + the quantum. To express the standard spacetime and energy-momentum variables, the various gauge fields, and the dynamical law in terms of these basics is nontrivial and would be a genuine unity.

The QND I use here borrows from the usual quantum field theory two fixed algebraic elements of $\mathbb{Q}$ labeled $\uparrow$, $\downarrow$, and the operator i, and retains the concept of a permanent dynamical law. All these seem arbitrary in QND, and so I expect that they are order parameters. It is known that the imaginary unit, as an order parameter $i = i(x)$ subject to variations $\delta i(x)$ preserving $i^2 = -1$, is a natural SU_2 Higgs field, making a massive charged photon-companion.

3. QUANTUM NETWORK DYNAMICS

3.1. Nonunitarity

I turn now to a quantum network dynamics in $\mathbb{Q}$. QND must sacrifice one of local relativity, superposition, unitarity, and local finiteness, for there is no finite-dimensional unitary representation of the Lorentz group. Unitarity is the unique correct sacrifice for the following reasons.

1. Unitarity expresses conservation of probability in time and is meaningful only for entities which persist in time, like stable particles. A unitary theory is less appropriate for the creation of the universe than for any other phenomenon that has occurred so far.
2. Unitary structure is nonlocal, involving an integral over a time slice.
3. Unitarity need not be postulated, but may be simulated in a nonunitary theory by a spontaneous breaking of the linear group, as a property of the condensed phase, like ferromagnetism.
4. $\mathbb{Q}$ is a nonunitary quantum theory, unlike the quantum set theory of Finkelstein *et al.* (1959). Like most theories, it may be interpreted

either as a phenomenological theory or a fundamental one. To use $\mathbb{Q}$ as a phenomenological theory, I provide a nonunitary form of the quantum principle, one not mentioning probability and valid for the usual quantum theory as well.

Quantum Principle. An input process is described by a qet and an output one by a dual qet, such that the transition is forbidden if the contraction is 0.

Probably we do not require probability formulas in the foundations of quantum theory. Probability is a poor man's set theory. It deals with highly special set variables of a large set of trials, those which are averages of individual variables over the set. $\mathbb{Q}$ can express these variables and more.

3.2. Intensionality

The usual quantum theory describes properties or classes with Hermitian operators, using the Hilbert space metric. We cannot do this in the nonunitary quantum theory of $\mathbb{Q}$. As a guide to the correct procedure, I return to $\mathbb{S}$.

To describe properties in $\mathbb{S}$, standard logical practice assumes extensionality, an isomorphism from (finite!) classes to sets, and represents classes or properties by elements of $\mathbb{S}$.

I emulate this in $\mathbb{Q}$; this is not yet standard practice in quantum logic. $\mathbb{Q}$ shall be both the algebras of quantum classes and quantum sets in one. We may read the symbol $\alpha \in \mathbb{Q}$ as a set, $\iota\alpha$ as the unit set or monad of α, $\iota\iota\alpha$ as the monad of the monad of α, and so forth. Then $\alpha\beta$ is the disjoint union. But we may also read each $\alpha \in \mathbb{Q}$ as a class or predicate, $\iota\alpha$ as the predicate of being α, $\iota\iota\alpha$ as the predicate of being the predicate of being α, and so forth. Then $\alpha\beta$ is the disjoint union. But we may also read each $\alpha \in \mathbb{Q}$ as a class or predicate, $\iota\alpha$ as the predicate of being α, $\iota\iota\alpha$ as the predicate of being the predicate of being α, and so forth. Then $\alpha\beta$ is the quantum correspondent of the Peirce partial disjunction $\alpha\text{POR}\beta$, and I write it as $\alpha\text{QOR}\beta$.

3.3. Chronons

QND relates dynamical entities of two kinds, events and chronons (Finkelstein, 1969), corresponding to vertices and directed edges of a graph, or states and transitions. In a provisional QND, I construct all events from the first event 1, two constant independent chronons $\delta_\Sigma = \uparrow, \downarrow$, spinor analogues of timelike future vectors or arrows, and a fixed operator i on $\mathbb{Q}$ playing the role of the complex imaginary, a superselection law; how superselection laws may arise from random phases is already understood.

The chronons $\delta = \uparrow, \downarrow$ may be regarded as dynamical impulses. In the "one-particle" part of the theory, the equation (1) giving the dynamical response β to impulse δ_Σ acting at event α takes the form

$$\alpha_\Sigma = \iota(\delta_\Sigma \alpha) \tag{2}$$

If there are not two, but N independent δ's, then N equations of the form (2) give the N successors α_Σ, $\Sigma = 1, \ldots, N$, of an event α, for N independent links $\delta_1, \ldots, \delta_N$. I call the $N+1$ events so related an N-ary node, or N-ode. An N-ode has the group SL_N, the special linear group on its N final events. Any collection of equations like (2) with possible identifications among the α's defines a quantum dynamical network.

4. VACUUM I

Vacuum I is the background for general relativity and the big bang. I now form a trial Vacuum I for QND. This part of the work is kinematical.

4.1. Quantum Equivalence Principle

The first thing I account for is the principle of equivalence, the validity of the Lorentz group of the macroscopic causual relation in the tangent space. The Lorentz group itself is not the group of any node, but its covering group SL_2 is, namely of a binary node. One reason to choose the Grassmann product $\vee$ as basic combinatorial operation (rather than, say, the Clifford product as earlier) is so that the two successors transform as a spinor under SL_2. In QND, events are related to their successors not by infinitesimal vectors as in $x' = x + dx$, but by finite spinors, and Einstein's equivalence principle follows from the following:

Quantum Equivalence principle. The successors of an event are two equivalent quantum entities with Fermi-Dirac statistics.

There are therefore two basic chronon qets, which I write as $\delta_\Sigma = \langle\Sigma| = \uparrow$, $\downarrow$. The two successors ψ_Σ of any event ψ are given by

$$\psi_\Sigma = \langle\langle\Sigma|\psi|$$

After Z chronons, a path from an arbitrary event ψ may arrive at any of the events $\psi_{\Sigma\ldots\Sigma'}$ with Z binary indices. I write (Σ) for such a sequence of Z indices of the type Σ. The process begins with an initial event $\psi = 1$, the null qet. Then the algebra KH of kinematic histories is generated by the paths defined inductively by

$$\psi_{\Sigma'(\Sigma)} = \langle\langle\Sigma'|\psi_{(\Sigma)}| \tag{3}$$

Each of the polyspinors $\psi_{(\Sigma)}$ describes not merely an endpoint, but an entire path, which may be developed from the endpoint by repeated debracketing.

4.2. Chronon Pair Formation

Having accounted for the group SL_2, I ask why the space-time manifold (actually, its tangent bundle) seems to be made of vectors, not spinors. Vectors are products of spinors and "antispinors" (complex conjugate spinors).

I infer that two fermionic chronons in sequence (3) form a pair described by a qet with the vectorial transformation law of $\langle\Sigma\Sigma^*|$.

It follows that the operator ι anticommutes with the operator i (is antilinear).

Then spinors and antispinors alternate in sequence in $\psi_{(\Sigma)}$. I write σ for the pair index $\Sigma\Sigma^*$. The module is described by a pair spinor $\langle\sigma|$ and the path with even Z has the form $\psi_{(\sigma)}$.

Thus, the basic module (3) of our trial vacuum has one initial and four final events. The actual vacuum may be made of various modules with probability amplitudes to be determined.

4.3. The Hypercubic Vacuum Lattice

Finally we must account for the macroscopic observable nature of space-time vectors. They are not quantum ψ vectors, states, describing quantum channels, but macroscopic objects, and serve as parameters or observables themselves.

Macroscopic classical objects are aggregations of microscopic quantum ones, but these ensembles may be incoherent (like the density field in the Thomas–Fermi model of the atom, which forgets quantum phases) or coherent (like the velocity field in superfluidity, which preserves quantum phases). In one case what emerges as the classical variable is a probability distribution ρ, in the other a probability amplitude distribution ψ. We may call the respective inverse problems (going from macroscopic theory to quantum) incoherent and coherent quantization, according as they start without or with quantum phases.

Canonical quantization begins from a theory without quantum phases; it is an incoherent quantization, extrapolating from high quantum numbers to low.

On the other hand, we make a correct theory of a Josephson junction potential $V(t)$, for example, not by canonically quantizing V, but by coherently quantizing it, which entails recognizing it as a quantum phase.

The space-time vector $dx^{\Sigma^*\Sigma}$ transforms not as an incoherent statistical operator $\rho_{\Sigma}^{\Sigma'}$, but as a coherent state vector of a pair. I therefore procede coherently.

Suppose that the algebra of dynamically allowed paths *DH* in Vacuum I is the symmetric subspace of the tensors $\psi_{(\sigma)}$. If $\{\sigma\}$ stands for a symmetric sequence of σ's, the collective index of a symmetric tensor, the quantum-space-time paths in Vacuum I have the form $\psi_{\{\sigma\}}$ with a symmetric sequence of $Z/2$ σ-indices. One may then identify the momentum operator ∂_σ with the Bose–Einstein creation operator ψ_σ on *DH*, and the coordinate operator x^σ with the dual Bose–Einstein annihilation operator on *DH*. With this identification the initial event 1 has definite space-time coordinates $x^\sigma = 0$ and indefinite energy-momentum, which is more plausible than any other linear combination. All physical time-space vectors v^s are regarded as macroscopic ψ vectors $v^{\Sigma^*\Sigma}$ of condensed aggregates of Σ–Σ^* pairs, present only in the cold phase, Vacuum I. The null sequence $\{\sigma\} = 1$ yields the ur-event $\psi_1 = 1$. The canonical commutation relations between p and x follow from algebra, not from the differential calculus of the continuum. The unitary structure based on classical Minkowski space-time emerges in the nonuniform limit of large chronon numbers.

This dynamics does not reside on a prior space-time. The space-time coordinates are operators on the prior space of dynamical histories.

This symmetry of *DH* means that two paths which differ only in the order of their elements or chronons lead to the same event. This specifies the topology of Vacuum I.

A $\psi_{\{\sigma\}}$ is defined by four integers giving the number of indices of the four kinds $\uparrow\uparrow^*$, $\uparrow\downarrow^*$, $\downarrow\uparrow^*$, $\downarrow\downarrow^*$; four occupation numbers, in other words. These basic events thus form a "sexidecimant" of the infinite four-dimensional hypercubical lattice. The events $\psi_{\Sigma\{\sigma\}}$ provide two interstitial events for each unit cell.

This lattice unites and extends the two-dimensional checkerboard of Feynman (1972, esp. pp. 168–169) and Feynman and Hibbs (1965), the quantum space-time of von Weizsäcker (1951, 1955), and the two-dimensional spherical spin network of Penrose (1971). [Penrose told me about his spin networks in 1959–60, though I did not take up the idea until Finkelstein (1968, 1969*a*,*b*).] A lattice with different elements but with a similar topology is derived from a continuum quantum space-time theory by Das (1989).

A simple action for the Dirac dynamics of a spin-1/2 quantum along the lines of Feynman's checkers game is given in Finkelstein (1989). It identifies the interstitial qets of the above lattice with fermion annihilation operators, which also anticommute and vanish on the vacuum. QND is more monistic than a unified field theory can be.

Such a quantum condensation seems required to account for the following features of standard physics, which would otherwise be incomprehensible in QND.

4.4. Supermobility

The law of inertia and momentum conservation are problematic in any discrete space-time, since the network is not invariant under translation. The corresponding momentum transfer in a crystal is an *Umklapp* process. Since we see no *Umklapp* in the vacuum up to enormous energies, E, we may be inclined to place an extremely low upper bound $<1/E$ on the fundamental cell size or chronon of the network. But this would be incorrect if an *Umklapp* requires a macroscopic number of coherent events. Then inertia is a macroscopic quantum effect. Since Newton's first law states that the mobility of a particle, usually defined as $(\partial[\text{force}]/\partial[\text{velocity}])^{-1}$ at zero velocity, is infinite in the vacuum, we may call the law of inertia *supermobility* to class it with the more recently discovered macroscopic quantum phenomena of superfluidity and superconductivity. Since the event pairing occurs between neighbors in space-time rather than momentum space, the vacuum in thermal equilibrium is presumably not a two-fluid system like liquid He II.

Spinors and pair spinors are complex, yet time space and gauge vectors are real. If this is a spontaneous breaking of gauge invariance as in superconductivity, then it is necessary to Hermitian-symmetrize the module $\langle\sigma|$ before forming the sequence $\psi_{\{\sigma\}}$.

4.5. Particle Symmetries

The internal particle symmetries all seem to be gauge symmetries. Gauge fields describe defects in the vacuum network, in the way that the Burgers vector does in a crystal. A path from gauge fields to topology is already rather well marked in a simplicial theory of space-time:

1. Every gauge field is the commutator of a gauge-covariant derivative operator with itself.
2. This derivative is a limit of the coboundary operator of cohomology.
3. The coboundary operator and its commutation relations express the topology of the complex.

If we can retrace this line of connections in QND, we will arrive at a purely topological theory of the gauge fields. Identifying the derivative operator with module creation operators corresponds to step 2.

Since we already have the network correspondent to spin, and none to charge or the other coupling constants, the first step is a theory of the transport of spin in the vacuum network, describing those defects in the network which produce torsion and curvature in space-time, presumably dislocations and disclinations, respectively. That would constitute a quantum theory of gravity.

To extend the gauge theory of nets from gravity to the other interactions, we have to assign defect structures to the known internal symmetry generators.

In the most immediate model of color SU_3 symmetry within network theory, each event supports not only the two "external" chronons already described, which link up into long paths of the macroscopic vacuum, but also three additional microscopic "internal" chronons that do not. This vacuum resembles a fur-covered checkerboard. Although the global structure of the network is four-dimensional, as if the nodes were binary, each node is actually a 5-ode. The color group then mixes the internal chronons. This discrete quantum version of Kaluza–Klein theory puts a heavy responsibility on the network dynamics, which must bind just three internal chronons to every two external ones, but QND suggests that such a structure actually exists. More generally, color ought to label a natural trio of distinguishable defects in the vacuum network which are isomorphic but not mixed by SL_2; I have not found such defects yet, except the internal chronons already described.

5. VACUUM II

This is the vacuum phase that precedes the manifold.

Evidently there is a causal ordering of the events in this phase as in any other. The events of $\mathbb{Q}$ are all partially ordered by the ancestral $\in^*$ of the membership relation $\in$ defined by the successor relation ι.

As we go back to higher temperatures, presumably the condensation of pairs into paths will break up first, and then the pairs themselves. I see no reason to suppose that Vacuum II exhibits any fixed dimensionality. Since the coordinate representation depends on the hypercubical lattice of Vacuum I, there is no natural concept of coordinate system for Vacuum II. Likely there are several distinct concepts of proper time consistent with the dynamical structure of Vacuum II which coincide for Vacuum I.

The binary node does not support time reversal T or parity P; it is made of two-component, irreversible (Weyl, chiral) spinor entities $\langle\Sigma|$, not reversible vector ones. There is no need to break P or T in QND; the problem is to create them. Vacuum I is organized so that, for example, each event has two immediate predecessors as well as two successors, which may be taken as the T transforms of each other. Vacuum II lacks this crystal-like organization, and P and T as well.

These symmetries will also be violated locally if sufficient energy is injected at a point of Vacuum I to induce a phase transition. I identify this breaking tentatively with the P and T violation that is observed in the weak interactions, since I know no other. Then, on dimensional grounds (unreliable in a theory with so many large dimensionless numbers) the critical temperature T_C and the chronon ƞ should be closer to $M_W \sim 10^2$ GeV, the mass of the W particle, than to the Planck mass $M_P \sim 10^{19}$ GeV. This suggests that balls of Vacuum II are produced in experiments today as well as in the First Flash. The experimental implications of this are not yet known.

6. THE FIRST FLASH

I call event 1 the First Flash in belated recognition of Peirce, who describes "the first stages of development, before time existed" thus:

> Out of the womb of indeterminacy we must say that there would have come something by the principle of Firstness, which we may call a flash. Then by the principle of habit there would have been a second flash. Though time would not yet have been, this second flash was in some sense after the first, because resulting from it.... We have no reason to think that even now time is quite perfectly continuous and uniform in its flow. (Peirce, 1890)

> We start, then, with nothing, pure zero. But this is not the nothing of negation.... The nothing of negation is the nothing of death, which comes *second* to, or after, everything. But this pure zero is the nothing of not having been born. There is no individual thing, no compulsion, outward or inward, no law. It is the germinal nothing, in which the whole universe is involved or foreshadowed.... It is boundless freedom. (Peirce, 1898)

Only one event in $\mathbb{Q}$ certainly has no predecessor, and that is the null set 1, which Peirce calls 0. The Big Bang is the later stage of cosmogony when the continuum concepts of general relativity apply. The First Flash is the lightning, the Big Bang is the thunder.

7. DYNAMICS

7.1. Quantum Dynamics

Time-slices and the concept of dynamical law as a one-parameter group of global transformations seem too global to be basic, except in a purely timelike one-dimensional world. But Heisenberg's original idea of dynamical law simply as a differential equation for the dynamical variables is satisfactorily local. Its generalization to networks is a collection of operator equations relating variables at neighboring events. We may use the global action principle of quantum mechanics to define such local equations.

A local dynamical theory has two algebras of variables: a kinematic algebra *KA* free of dynamical equations, and a dynamical algebra *DA* whose variables obey the dynamical equations. The dynamical algebra is a quotient of the kinematical one modulo the dynamical equations *DE*:

$$DA = KA/DE$$

Quantum theories whose ψ vectors represent a process that happens on one time-slice may be called synchronic. They compress an input process that may actually be distributed over time into one initial instant. They assign input and outputs to different linear spaces, thus blocking their linear superposition and tacitly positing a temporal superselection rule, an artifact of the c/q partition.

Theories whose ψ vectors represent histories of what happens in all of space-time may be called diachronic. The Schwinger and Feynman quantum action principles may be formulated as diachronic theories. In the Schwinger (1970) source theory, a source is an element of a single linear space *S*, the source space, and describes both input and output (i/o) processes over the entire experimental space-time region, allowing their superposition and lifting the superselection law between them. Sources are "superlocal" in that sources separated by timelike intervals, just like those separated by spacelike intervals, represent independent choices of the experimenter and are not related by dynamical equations. The distinction between input and output is made within *S* purely on the basis of the sign of the frequency.

Each dual vector assigns a transition amplitude to each source. It therefore expresses a dynamical law or force law, and is called a field for short. I write *F* for an algebra of fields dually isomorphic to *S*, such that *F* and *S* are included in each other's duals. Each field determines a system of propagators. One field $\langle \text{vac}|$ determines the vacuum propagators and may equally well be called the vacuum or the law of nature. It stores the forms of all the phenomenological interactions, and the charges, masses, and other coupling constants of the experimental quanta.

Feynman's path amplitude represents the dynamical law and is therefore an element of *F*, not *S*, and defines the vacuum.

Since histories and variables assign numerical values to each other, they are categorically dual. The algebra of dynamically allowed histories *DH* is therefore a subalgebra, the dual concept to a quotient algebra, of the algebra of kinematical histories *KH*; namely, the subalgebra of kinematical histories that fulfill the dynamical equations of *DE*:

$$DH = KH \backslash DE$$

For Vacuum I the algebras KH and DH are given in Section 4.3. To specify the dynamical equations DE, in Finkelstein (1989) I identify the creation operator $\psi_{\Sigma\{\sigma\}}$ with the field operator for the space-time point labeled by $\{\sigma\}$ and the spin eigenvalue labeled by Σ. It is then straightforward to translate the Weyl and Dirac particles, for example, into the algebraic language of $\mathbb{Q}$. They seem to gain in translation, becoming locally finite, without ultraviolet catastrophes.

7.2. Origin of Dynamics

In diachronic quantum descriptions, the dynamical law is merely one element of the space F of fields. Such a preordained dynamical law or vacuum seems unlikely to many, including me. Therefore I speculate here on an autonomous dynamics, one that does not add a second story to Fig. 2b. Here I venture beyond my elementary mathematical models.

A path to an autonomous dynamics was sketched by L. Susskind. One starts from a random dynamics [in the sense of Nielsen and Ninomiya (1989)] and renormalizes. The long-time dynamics of a system averages the random dynamics of the universe over all the variables outside the system and over the short-time variables of the system itself, and washes out vastly more variables than it retains. In the vicinity of a critical point, an unstable fixed point of the renormalization group dominates the average.

An evolving random quantum topology fits Peirce's doctrines of Tychism and Synechism, and Wheeler's principle that "The only law is the Law of Large Numbers." The main difficulties I have had with this principle until now, and my present hopes for their resolution, are:

1. I expected a random space-time to result in incoherent space-time propagation. But now the quantum condensation to Vacuum I may provide the necessary coherence.

2. In field theory the space DH of all possible action principles is unmanageably infinite, and to average we must choose a probability measure; one might as well choose a dynamics. In QND, however, $DH \subset \mathbb{Q}$ is finitely generated, and it is natural to average over all dynamics in order of increasing rank (power of ι). No further measure is required.

The natural coarse-graining transformation is the substitution $\iota^n \to \iota$, which regards nth successors as immediate successors, and so changes the scale of space-time by a factor n. The module constructed in Section 4.2 is only the first vector network. Near critical points, iterated coarse-graining replaces this simple module by indefinitely complex vector modules able to carry much more information. Schrödinger taught us to think of organic structure as encoded in an "aperiodic crystal"; perhaps we must learn to think of dynamical structure in the same way.

8. SUMMARY

The salient inferences about the creation from QND are:

1. The origin of the universe is a single quantum point event, the First Flash.
2. Quantum-space-time is a quantum combination of quantum elements of spin 1/2 with Fermi–Dirac statistics.
3. These spin-1/2 quantum elements are microscopic arrows of time, "chronons," without time reversal.
4. Chronons unite with their complex conjugates in sequential pairs to form quantum-space-time vectors.
5. Vectors unite in long sequences to form quantum-space-time paths.
6. Spacetime points are equivalence classes of such paths.
7. Vacuum I, the present ambient space-time, with its Minkowski chronometry, is a critical phenomenon, a Bose condensation of chronon pairs into a hypercubical lattice of basic states.
8. Defects in the QND Vacuum I lattice give rise to the gauge fields.
9. Vacuum II, the phase preceding Vacuum I, is a chaotic, but nonetheless causally ordered network of events.

ACKNOWLEDGMENTS

This paper is based in part on work supported by National Science Foundation Grant PHY-8410463, and by the Georgia Tech Foundation Award 24AF63. During this writing I am pleasurably indebted to Roger Penrose and the Mathematical Institute, to Harvey Brown and the Philosophy Department, and to Wolfson College, Oxford University, for their hospitality, and to Georgia Institute of Technology for sabbatical leave and support. I thank Raphael Sorkin and John Barrett for useful discussions and Shlomit Finkelstein for many helpful criticisms of the manuscript as well.

REFERENCES

Barrett, J. W. (1989). Private communication.

Bombelli, L., Lee, J., Meyer, D., and Sorkin, R. (1987). Spacetime as a locally finite causal set, *Physical Review Letters* **59**, 521–524.

Chew, G. F., and Stapp, H. P. (1989). Pre-geometrical quantum cosmological evolution, Preprint LBL-27096, Lawrence Berkeley Laboratory.

Das, A. (1989). Quantized phase space and relativistic quantum theory, *Nuclear Physics B*, (Proceedings Supplement), **6**.

Einstein, A. (1936). Physics and reality, *Journal of the Franklin Institute*, **221**, 378.

Feynman, R. P. (1972). The development of the space-time view of quantum electrodynamics, in *Nobel Lectures—Physics—1963-1970*, Elsevier.

Feynman, R. P., and Hibbs, A. R. (1965). *Quantum Mechanics Via Path Integrals*, Chapter 2, McGraw-Hill, New York.

Finkelstein, D. (1958). Past-future asymmetry of the gravitational field of a point particle, *Physical Review*, **110**, 965-977.

Finkelstein, D. (1968). Matter, space and logic, in *Boston Studies in the Philosophy of Science*, Vol. 5, R. S. Cohen and M. W. Wartofsky, eds., pp. 199-215.

Finkelstein, D. (1969). Space-time code, *Physical Review*, **184**, 1261-1271.

Finkelstein, D. (1978). Process philosophy and quantum dynamics, in *Physical Theory as Logico-Operational Structure*, C. A. Hooker, ed., D. Reidel, Dordrecht.

Finkelstein, D. (1987). Coherent quantum logic, *International Journal of Theoretical Physics*, **26**, 109-129.

Finkelstein, D. (1988). "Superconducting" causal nets, *International Journal of Theoretical Physics*, **27**, 473-519.

Finkelstein, D. (1989). Quantum net dynamics, *International Journal of Theoretical Physics*, **28**, 441-468.

Finkelstein, D., Jauch, J. M., and Speiser, D. (1959). Quaternion quantum mechanics III, European Center for Nuclear Research, Geneva, Report 59-17 [reprinted in *The Logico-Algebraic Approach to Quantum Mechanics, II*, C. A. Hooker, ed., D. Reidel, Dordrecht (1979)].

Heisenberg, W. (1958). *Physics and Philosophy*, p. 46.

Misner, C., Thorne, K., and Wheeler, J. A. (1973). *Gravitation*, Freeman, San Francisco.

National Research Council (1986*a*). *Physics through the* 1990's. *Elementary-Particle Physics*, p. 97, National Academy Press.

National Research Council (1986*b*). *Physics through the 1990's. Gravitation, Cosmology, and Cosmic Ray Physics*, p. 74, National Academy Press.

Nielsen, H. B., and Ninomiya, M. (1989). *Nuclear Physics B*, **141**, 153.

Peirce, C. S. (1868*a*). On an improvement in Boole's calculus of logic, *Proceedings of the American Academy of Arts and Sciences*, **7**, 250-261. Peirce, C. S. (1868*b*). Upon the logics of mathematics, *Proceedings of the American Academy of Arts and Sciences*, **7**, 402-412.

Peirce, C. S. (ca. 1890). *A guess at the riddle*, Manuscript, Harvard Library Collection [reprinted in Peirce (1931-1953), Vol. 1, Paragraph 412].

Peirce, C. S. (1898). *The logic of events*, Manuscript, Harvard University Collection [reprinted in Peirce (1931-1953), Vol. 6, Paragraph 217].

Peirce, C. S. (1931-1953). *The Collected Papers of Charles Sanders Peirce*, Vols. I-VIII, Harvard University Press, Cambridge, Massachusetts.

Penrose, R. (1971) Angular momentum: An approach to combinatorial space-time, in *Quantum Theory and Beyond*, T. Bastin, ed., Cambridge University Press, Cambridge.

Ruffini, R., and Wheeler, J. A. (1970). Collapse of wave to black hole, *Bulletin of the American Physical Society*, **15**, 76.

Schwinger, J. (1970). *Particles, Sources, and Fields*, Addison-Wesley.

Susskind, L. (ca. 1970). Private communication.

Tryon, E. P. (1973). Is the universe a vacuum fluctuation?, *Nature*, **246**, 396-397.

Von Weizsäcker, C. F. (1951). *Naturwissenschaften*, **38**, 553.

Von Weizsäcker, C. F. (1955). *Naturwissenschaften*, **42**, 548.

Wheeler, J. A. (1973). In *The Physicist's Conception of Nature*, J. Mehra, ed., D. Reidel, Dordrecht.